PRE COMPITINO 24/05/2013

PARADOSSO

NON VALIDA

$$\dfrac{\dfrac{\dfrac{\dfrac{\neg B \leftrightarrow B, A \vdash C}{\neg B \leftrightarrow B, A \vdash A \to C}\ \to D}{\neg B \leftrightarrow B \vee A \vdash A \to C}\ \vee\text{-}S \qquad \neg B \leftrightarrow B \vee A, C \to B \vdash}{\neg B \leftrightarrow B \vee A, (A \to C) \to (C \to B) \vdash}\ \to S}{(\neg B \leftrightarrow B) \vee A \vdash \neg((A \to C) \to (C \to B))}\ \neg\text{-}D$$

$$\models (\neg B \leftrightarrow B) \leftrightarrow \perp$$

CONTROMODELLO

$A=1 \quad C=0$

MODELLO

$A=1$

$C=1$

$B=0$

$$\underbrace{\perp \vee A}_{1} \vdash \underbrace{\neg\underbrace{(\underbrace{(A \to C)}_{1} \to \underbrace{(C \to B)}_{0})}_{0}}_{1}$$

QUINDI SODDISFACIBILE PER $A=1$, $C=1$, $B=0$

②

VALIDA

$=\text{-ax}$

$\vdash w=w$ $\quad = \text{-S}_v \quad$ $\bot\text{-ax}$

$w=x \vdash x=w \qquad \bot \vdash x=w$ $\quad \vee\text{-S}$

$w=x \vee \bot \vdash x=w$ $\quad \rightarrow D$

$\vdash w=x \vee \bot \rightarrow x=w$ $\quad \exists\text{-D}_v$

$\vdash \exists z\,(w=x \vee \bot \rightarrow x=z)$ $\quad \forall\text{-D} \;\; x \notin VL$

$\vdash \forall y\, \exists z\,(w=y \vee \bot \rightarrow y=z)$ $\quad \exists\text{-D}_v$

$\vdash \exists x\, \forall y\, \exists z\,(x=y \vee \bot \rightarrow y=z)$

③

VALIDO

$=\text{-ax}$

$\vdash x=x$ $\quad =\text{-S}_v \qquad \text{ax-id}$

$x=w \vdash w=x \qquad x=a \vdash x=a$ $\quad \neg\text{-S}$

$x=w,\, w\neq x \vdash \qquad x=a,\, x\neq a \vdash$ $\quad \text{sc-sx}$

$w\neq x,\, x=w \vdash \qquad x\neq a,\, x=a \vdash$ $\quad \neg\text{-D}$

$w\neq x \vdash x\neq w \qquad x\neq a \vdash x\neq a$ $\quad \exists\text{-D}_v$

$w\neq x \vdash \exists z\, x\neq z \qquad x\neq a \vdash \exists z\, x\neq z$ $\quad \vee\text{-S}$

$w\neq x \vee x\neq a \vdash \exists z\, x\neq z$ $\quad \rightarrow D$

$\vdash w\neq x \vee x\neq a \rightarrow \exists z\, x\neq z$ $\quad \forall\text{-D} \;\; x \notin VL$

$\vdash \forall y\,(w\neq y \vee y\neq a \rightarrow \exists z\, y\neq z)$ $\quad \forall\text{-D} \;\; w \notin VL$

$\vdash \forall x\, \forall y\,(x\neq y \vee y\neq a \rightarrow \exists z\, y\neq z)$

④

VALIDA

$$\dfrac{\dfrac{\overset{\neg\text{-ax}_{sx1}}{B(x), \neg B(x) \vdash \forall w(\_)}}{B(x) \,\&\, \neg B(x) \vdash \forall w(\_)}\ \&\text{-S}}{\exists w\, (B(w) \,\&\, \neg B(w)) \vdash \forall w\, (A(w) \,\&\, \neg A(w))}\ \exists\text{-S}\ x \notin VL$$

⑤

NON VALIDA

$$\dfrac{}{B(w) \vdash \forall w\, (A(w) \vee \bot)}$$

CONTROMODELLO : D:N

$B(x)^D(d) = 1$ SEMPRE

$A(x)^D(d) = 0$ SEMPRE

QUINDI PER $d \in D$:

$(\forall w\, (A(w) \vee \bot))^D(d) = 0$

QUINDI PER $d, d' \in D$:

$(B(w) \to (A(x) \vee \bot))^D(d, d') = 0$

MODELLO

$D = \{PLUTO\}$

$B(w)^D(d) = 0$ SEMPRE

QUINDI PER OGNI $d \in D$

$(B(w) \to \forall x\, (A(x) \vee \bot))^D(d) = 1$

SODDISFACIBILE

(6)

VALIDA

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\neg\text{-ax}_{sx1}}{A(x), B(w), \neg A(x) \vdash}\ \forall\text{-S}_v}{A(x), B(w), \forall w \neg A(w) \vdash}\ \neg\text{-D}}{A(x), B(w) \vdash \neg \forall w \neg A(w)}\ \&\text{-S}}{A(x) \& B(w) \vdash \neg \forall w \neg A(w)}\ \exists\text{-S}\ x \notin VL}{\exists x (A(x) \& B(w)) \vdash \neg \forall w \neg A(w)}\ \to\text{-D}}{\vdash \exists x (A(x) \& B(w)) \to \neg \forall w \neg A(w)}$$

(7)

NON VALIDA

$$\dfrac{\dfrac{\dfrac{A(x) \vdash \neg\neg A(w) \vee \bot}{A(x) \vdash \forall x (\neg\neg A(x) \vee \bot)}\ \forall\text{-D}\ w \neq VL}{\exists x A(x) \vdash \forall x (\neg\neg A(x) \vee \bot)}\ \exists_S\ x \neq VL}{\vdash \exists x A(x) \to \forall x (\neg\neg A(x) \vee \bot)}\ \to\text{-D}$$

CONTROMODELLO

$D: \{\text{MINNI}, \text{TOPOLINO}\}$

$A(x)^D(d) = 1$ SSE x è FEM.

QUINDI PER $d \in D$:

$(\exists x\, A(x))^D(d) = 1$ PERCHÉ

$A(x)^D(\text{MINNI}) = 1$ e

$(\forall x (\neg\neg A(x) \vee \bot))^D(d) = 0$ PERCHÉ

$(\forall x (\neg\neg A(x) \vee \bot))^D(\text{TOPOLINO}) = 0$

QUINDI PER $d \in D$

$(\exists x\, A(x) \to \forall x (\neg\neg A(x) \vee \bot))^D(d) = 0$

MODELLO

$D = \{\text{MINNI}\}$

$(A(x))^D(d) = 1$ SEMPRE

$(\exists x\, A(x) \to \forall x (\neg\neg A(x) \vee \bot))^D(d) = 1$

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VALIDO

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{ax\text{-}id}{A(w), A(z) \vdash A(z), C(x)}}{A(w), \forall y A(y) \vdash A(z), C(x)}\ \forall\text{-}S\vee}{\forall y A(y), A(w) \vdash A(z), C(x)}\ sc\text{-}sx}{\forall y A(y), A(w) \vdash A(z) \vee C(x)}\ \vee\text{-}D}{\forall y (A(y)) \,\&\, A(w) \vdash A(z) \vee C(x)}\ \&\text{-}S}{\forall y (A(y)) \,\&\, A(w) \vdash \forall w (A(w) \vee C(x))}\ \forall\text{-}D\ z \neq VL$$

9 $\forall x (C(x) \to F(x)) \to \forall x (P(x) \to S(x))$,
$\forall x (C(x) \to \neg E(x)) \to \forall x (P(x) \to L(x)) \vdash$
$\forall x (C(x) \to F(x) \vee \neg E(x))$

NON VALIDA

CONTROMODELLO

$D = \{PLUTO\}$

$C(x)^D(d) = 1$ SEMPRE
$F(x)^D(d) = 0$ SEMPRE
$E(x)^D(d) = 1$ SEMPRE
ALTRI A PIACERE

PER OGNI d:
$(\forall x (C(x) \to F(x) \vee \neg E(x))^D(d) = 0$

PER OGNI d:

$C(x)^D(d) \to F(x)(d) \vee E(x)^D(d) = 0$

$\forall x (C(x) \to F(x)) \to \forall x (P(x) \to S(x)) = 1$ NON VALIDA
$\forall x (C(x) \to \neg E(x)) \to \forall x (P(x) \to L(x)) = 1$

(10)

VALIDA

$=$-ax

$I(v,G), I(l,G), \exists x(-) \vdash v = v$

$=$-Sv

$I(v,G), I(l,G), \exists x(-), v = l \vdash l = v$

$\neg$-S

ax-id

ax-id

$I(v,G), I(l,G), l \neq v, \exists x\, I(x,G) \vdash I(v,G)$ $\quad$ $I(v,G), I(l,G), l \neq v, \exists x(-) \vdash I(l,G)$ $\quad$ $I(v,G), I(l,G), \exists x(-), v = l, l \neq v \vdash$

&-D $\quad$ sc sx

$I(v,G), I(l,G), l \neq v, \exists x\, I(x,G) \vdash I(v,G) \,\&\, I(l,G)$ $\quad$ $I(v,G), I(l,G), l \neq v, \exists x\, I(x,G), v = l \vdash$

$\to$-S

$I(v,G), I(l,G), l \neq v, \exists x\, I(x,G), I(v,G) \,\&\, I(l,G) \to v = l \vdash$

$\forall$-Sv

$I(v,G), I(l,G), l \neq v, \exists x\, I(x,G), \forall y_2 (I(v,G) \,\&\, I(y_2,G) \to v = y_2) \vdash$

$\forall$-Sv

$I(v,G), I(l,G), l \neq v, \exists x\, I(x,G), \forall y_1 \forall y_2 (I(y_1,G) \,\&\, I(y_2,G) \to y_1 = y_2) \vdash$

&-S

$I(v,G), I(l,G), l \neq v, \exists x\, I(x,G) \,\&\, \forall y_1 \forall y_2 (I(y_1,G) \,\&\, I(y_2,G) \to y_1 = y_2) \vdash$

$\neg$-D

$I(v,G), I(l,G), l \neq v \vdash \neg(\exists x\, I(x,G) \,\&\, \forall y_1 \forall y_2 (I(y_1,G) \,\&\, I(y_2,G) \to y_1 = y_2$

(11)

VALIDA

ax-id

$I(v,G), \exists x\, I(-), I(v,G), l = v \vdash l = v$

$\neg$-S

ax-id

... CON E SOPRA

$I(v,G), \exists x\, I(-), I(v,G), l = v, l \neq v \vdash$

sc sx

$l \neq v, I(l,G), \exists x\, I(-), I(v,G) \vdash I(l,G) \,\&\, I(v,G)$ $\quad$ $l \neq v, I(v,G), \exists x\, I(-), I(v,G), l = v \vdash$

$\to$-S

$l \neq v, I(l,G), \exists x\, I(-), I(v,G), I(l,G) \,\&\, I(v,G) \to l = v \vdash$

sc sx

$l \neq v, I(l,G), \exists x\, I(-), I(l,G) \,\&\, I(v,G) \to l = v, I(v,G) \vdash$

$\neg$-D

$l \neq v, I(l,G), \exists x\, I(-), I(l,G) \,\&\, I(v,G) \to l = v \vdash \neg I(v,G)$

$\forall$-Sv

$l \neq v, I(l,G), \exists x\, I(-), \forall y_2 (I(l,G) \,\&\, I(y_2,G) \to l = y_2) \vdash \neg I(v,G)$

$\forall$-Sv

$l \neq v, I(l,G), \exists x\, I(x,G), \forall y_1 \forall y_2 (I(y_1,G) \,\&\, I(y_2,G) \to y_1 = y_2) \vdash \neg I(v,G)$

&-S

$l \neq v, I(l,G), \exists x\, I(x,G) \,\&\, \forall y_1 \forall y_2 (I(y_1,G) \,\&\, I(y_2,G) \to y_1 = y_2) \vdash \neg I(v,G)$

sc sx

$\exists x\, I(x,G) \,\&\, \forall y_1 \forall y_2 (I(y_1,G) \,\&\, I(y_2,G) \to y_1 = y_2), l \neq v, I(l,G) \vdash \neg I(v,G)$

$\to$-D

$\exists x\, I(x,G) \,\&\, \forall y_1 \forall y_2 (I(y_1,G) \,\&\, I(y_2,G) \to y_1 = y_2), l \neq v \vdash I(l,G) \to \neg I(v,G)$

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NON VALIDA

$$\dfrac{\dfrac{\dfrac{\dfrac{\vdash T}{T \to (C \to \neg S) \vdash T}\ \to_S}{T \to (C \to \neg S),\ T \to (\neg C \to S) \vdash T}\ \to_S}{T \to (C \to \neg S),\ T \to (\neg C \to S),\ T \to S \vee C \vdash}\ \to_S}{T \to (C \to \neg S),\ T \to (\neg C \to S) \vdash \neg(T \to S \vee C)}\ \neg\text{-D}$$

CONTROMODELLO

$T = 0$

MODELLO

$T = 1$

$C = 0$

$S = 0$

$1 \to 1,\ 1 \to 1 \vdash \neg(1 \to 0)$

$1 \vdash 1$ ✓

SODISFACIBILE

è NELL'ESAME!

13

$$\forall x\,(P(x)\ \&\ C(x) \rightarrow P(x)\ \&\ U(x)),$$
$$P(x)\ \&\ \neg U(x) \vdash$$
$$P(x)\ \&\ \neg C(x)$$

VALIDA

$$\dfrac{\dfrac{}{P(x), U(x), P(x), \neg U(x) \vdash P(x)\ \&\ \neg C(x)}\ \neg\text{-ax sx1}}{P(x), U(x), P(x)\ \&\ \neg U(x) \vdash P(x)\ \&\ \neg C(x)}\ \&\text{-S}$$

$$\dfrac{P(x), U(x), P(x)\ \&\ \neg U(x) \vdash P(x)\ \&\ \neg C(x)}{P(x)\ \&\ \neg U(x), P(x), U(x) \vdash P(x)\ \&\ \neg C(x)}\ \text{sc sx}$$

$$\dfrac{P(x)\ \&\ \neg U(x), P(x), U(x) \vdash P(x)\ \&\ \neg C(x)}{P(x)\ \&\ \neg U(x), P(x)\ \&\ U(x) \vdash P(x)\ \&\ \neg C(x)}\ \&\text{-S}$$

$*^{1}$ riporto sotto

$$\dfrac{}{P(x)\ \&\ \neg U(x) \vdash P(x)\ \&\ C(x), P(x)\ \&\ \neg C(x)}\ \&\text{-}$$

$$\dfrac{P(x)\ \&\ \neg U(x), P(x)\ \&\ U(x) \vdash P(x)\ \&\ \neg C(x) \qquad P(x)\ \&\ \neg U(x) \vdash P(x)\ \&\ C(x), P(x)\ \&\ \neg C(x)}{P(x)\ \&\ \neg U(x), P(x)\ \&\ C(x) \rightarrow P(x)\ \&\ U(x) \vdash P(x)\ \&\ \neg C(x)}\ \rightarrow\text{-S}$$

$$\dfrac{P(x)\ \&\ \neg U(x), P(x)\ \&\ C(x) \rightarrow P(x)\ \&\ U(x) \vdash P(x)\ \&\ \neg C(x)}{P(x)\ \&\ \neg U(x), \forall x\,(P(x)\ \&\ C(x) \rightarrow P(x)\ \&\ U(x)) \vdash P(x)\ \&\ \neg C(x)}\ \forall\text{-S}_{v}$$

$$\dfrac{P(x)\ \&\ \neg U(x), \forall x\,(P(x)\ \&\ C(x) \rightarrow P(x)\ \&\ U(x)) \vdash P(x)\ \&\ \neg C(x)}{\forall x\,(P(x)\ \&\ C(x) \rightarrow P(x)\ \&\ U(x)), P(x)\ \&\ \neg U(x) \vdash P(x)\ \&\ \neg C(x)}\ \text{sc sx}$$

$$\dfrac{\dfrac{}{P(x), \neg U(x) \vdash P(x), C(x)}\ \text{ax id} \qquad \dfrac{}{P(x), \neg U(x) \vdash \neg C(x), C(x)}\ \neg\text{-ax dx2}}{P(x), \neg U(x) \vdash P(x)\ \&\ \neg C(x), C(x)}\ \&\text{-D}$$

$$\dfrac{P(x), \neg U(x) \vdash P(x)\ \&\ \neg C(x), C(x)}{P(x), \neg U(x) \vdash C(x), P(x)\ \&\ \neg C(x)}\ \text{sc dx}$$

$$\dfrac{\dfrac{}{P(x), \neg U(x) \vdash P(x), P(x)\ \&\ \neg C(x)}\ \text{ax id} \qquad P(x), \neg U(x) \vdash C(x), P(x)\ \&\ \neg C(x)}{P(x), \neg U(x) \vdash P(x)\ \&\ C(x), P(x)\ \&\ \neg C(x)}\ \&\text{-D}$$

$$\dfrac{P(x), \neg U(x) \vdash P(x)\ \&\ C(x), P(x)\ \&\ \neg C(x)}{P(x)\ \&\ \neg U(x) \vdash P(x)\ \&\ C(x), P(x)\ \&\ \neg C(x)}\ \&\text{-S}$$

(14)

VALIDA

$$\dfrac{\dfrac{\dfrac{\overset{ax\text{-}id}{C(w) \vdash C(w), \neg\forall y(\ldots), B(w)} \qquad \overset{\neg\text{-}ax_{dx1}}{C(w) \vdash \forall y A(y,w), \neg\forall y A(y,w), B(w)}}{C(w) \vdash C(w) \,\&\, \forall y A(y,w), \neg\forall y A(y,w), B(w)}\ \&\text{-}D \qquad \overset{ax\text{-}id}{C(w), B(w) \vdash \neg\forall y A(y,w), B(w)}}{C(w), C(w) \,\&\, \forall y A(y,w) \to B(w) \vdash \neg\forall y A(y,w), B(w)}\ \to S$$

$$\dfrac{C(w), C(w) \,\&\, \forall y A(y,w) \to B(w) \vdash \neg\forall y A(y,w), B(w)}{C(w) \,\&\, \forall y A(y,w) \to B(w), C(w) \vdash \neg\forall y A(y,w), B(w)}\ sc\text{-}sx$$

$$\dfrac{C(w) \,\&\, \forall y A(y,w) \to B(w), C(w) \vdash \neg\forall y A(y,w), B(w)}{C(w) \,\&\, \forall y A(y,w) \to B(w), C(w) \vdash \neg\forall y A(y,w) \vee B(w)}\ \vee\text{-}D$$

$$\dfrac{C(w) \,\&\, \forall y A(y,w) \to B(w), C(w) \vdash \neg\forall y A(y,w) \vee B(w)}{C(w) \,\&\, \forall y A(y,w) \to B(w) \vdash C(w) \to \neg\forall y A(y,w) \vee B(w)}\ \to D$$

$$\dfrac{C(w) \,\&\, \forall y A(y,w) \to B(w) \vdash C(w) \to \neg\forall y A(y,w) \vee B(w)}{\forall x (C(x) \,\&\, \forall y A(y,x) \to B(x)) \vdash C(w) \to \neg\forall y A(y,w) \vee B(w)}\ \forall\text{-}Sv$$

$$\dfrac{\forall x (C(x) \,\&\, \forall y A(y,x) \to B(x)) \vdash C(w) \to \neg\forall y A(y,w) \vee B(w)}{\forall x (C(x) \,\&\, \forall y A(y,x) \to B(x)) \vdash \forall x (C(x) \to \neg\forall y A(y,x) \vee B(x))}\ \forall\text{-}D \ (w \notin VL)$$

(15)

$$\exists x ((C(x) \,\&\, \forall y A(\cancel{y},\cancel{x})) \,\&\, \neg B(x)) \vdash \forall x (C(x) \to (B(x) \,\&\, \forall y A(y,x)) \vee \neg B(x)$$

NON VALIDA

$$\dfrac{\dfrac{\dfrac{\vdash C(x) \,\&\, \forall y A(y,x)}{\vdash \exists x ((C(x) \,\&\, \forall y A(y,x)) \,\&\, \neg B(x))}\ \exists\text{-}D}{\exists x ((C(x) \,\&\, \forall y A(y,x)) \,\&\, \neg B(x)) \to (\forall x (C(x) \to B(x) \,\&\, \forall y A(y,x)) \vee \neg B(x)) \vdash}\ \to S}{\vdash \neg(\exists x ((C(x) \,\&\, \forall y A(y,x)) \,\&\, \neg B(x)) \to (\forall x (C(x) \to (B(x) \,\&\, \forall y A(y,x)) \vee \neg B(x))}\ \neg\text{-}D$$